J. Fluid Mech. (2002), *vol.* 450, *pp.* 408–410. © 2002 Cambridge University Press 408
DOI: 10.1017/S0022112001007522 Printed in the United Kingdom

BOOK REVIEW

An Introduction to Magnetohydrodynamics. By P. A. Davidson. Cambridge University Press, 2001. 431 pp. ISBN 0 521 79149 9. £70.00 (hardback); ISBN 0 521 79487 0. £24.95 (paperback).

Magnetohydrodynamics encompasses a wide range of physical disciplines which include astro- and geo-physical fluid dynamics, laboratory and industrial applications, as well as the elusive search for effective plasma confinement. Each discipline has its own character. It is therefore a considerable challenge to write an introductory text which captures the flavour of all these topics and to a large extent Davidson's splendid book achieves that goal. It is written in a spirited style which conveys his enthusiasm for the subject, while there are a number of historical anecdotes which help to bring the subject alive. The book restricts attention to incompressible electrically conducting fluids and is divided into two parts. Part A deals with basic MHD theory while Part B focuses attention on the author's particular research interest of applications to engineering and metallurgy. The topics selected are well chosen; though the suggested astro- and geo-physical MHD textbooks on p. 423 are somewhat minimal, an assessment that may simply be a reflection of this reviewer's prejudices. The physical content of the material is stressed and clearly explained. Nevertheless, the mathematics and accompanying description is compact so that a student attempting to follow the details may find it challenging in places. All in all, however, this is an excellent book, which provides a refreshing introduction and a welcome addition to the MHD literature.

Much of the early development in Part A follows conventional lines. Chapters 1 and 2 introduce the Pre-Maxwell equations, Ohm's law for a moving conductor and the Lorentz force. Chapter 3 begins by developing basic ideas associated with viscous flow governed by the Navier–Stokes equations. The nature of large Reynolds number flows is stressed, including boundary layer theory and the Prandtl–Batchelor theory for closed streamline flows in the largely inviscid regions exterior to the boundary layers. The analogy is drawn between the vorticity equation and the magnetic induction equation. The similarity of vortex/magnetic field line stretching (Helmholtz's laws) and diffusion is discussed in relation to the size of the Reynolds (Re)/magnetic Reynolds (R_m) number. Applications to swirling flows are emphasized with discussion of Karman and Ekman layers. Finally the Maxwell stress is introduced and the notions of magnetic pressure and tension are explained.

Chapter 4 takes the kinematic ideas further. In the case of a perfectly conducting fluid, Alfvén's theorem (also conservation of magnetic helicity) is discussed with application to magnetic flux tubes in the Sun; here *Solar Magnetohydrodynamics* by E. R. Priest, Riedel (1982) ought to be a suggested book. The treatment of this application is by necessity a little superficial as a comprehensive discussion involves magnetic buoyancy as described in *Cosmical Magnetic Fields* by E. N. Parker, Oxford University Press (1979). Since that mechanism depends on compressibility, it lies outside the scope of this book. In the case of fluids with finite electrical conductivity specific models involving advection-diffusion are described which also illustrate the concept of magnetic flux expulsion. Magnetic field line reconnection is mentioned briefly; for further reading on this topic *Magnetic Reconnection MHD Theory and*

Applications by E. R. Priest & T. Forbes, Cambridge University Press (2000) would be appropriate.

Chapter 5 is devoted to low-R_m flows with the subject matter focused towards the later metallurgic applications of Part B. The main theme concerns fluid flows permeated by uniform applied magnetic fields $\boldsymbol{B}_0$, while the thrust of the development is to stress the tendency of motion to approach two-dimensionality independent of the coordinate parallel to $\boldsymbol{B}_0$. There are nice examples involving jets and vortices. MHD-turbulence is introduced briefly with emphasis on angular momentum, whose conservation is a recurring theme throughout the book. Steady Bénard convection and natural convection are also considered briefly. Magnetic stirring by rotating $\boldsymbol{B}_0$ about an orthogonal axis is investigated, as is the axisymmetric flow generated by current injection from a source of finite size. The chapter ends with a brief discussion of Hartmann layers, which to me lacks the depth of study that this topic deserves. Duct flow contains physical ingredients essential for a proper understanding of MHD, especially the role of the electrical boundary conditions. Whether currents are returned largely in the fluid or via the boundaries is vital and strongly influences the mass flux for a given pressure gradient, particularly at large Hartmann number. Ironically these issues are examined comprehensively in an out of print book with the identical title by P. H. Roberts, Longmans (1967). Still, the author has adopted the point of view that such in-depth study of duct flow problems is best left to other engineering books such as *Magnetohydrodynamics* by R. Moreau, Kluwer (1990) and the new book *Magnetofluiddynamics in Channels and Containers* by U. Müller & L. Bühler, Springer (2001). Let me stress again, however, that the electrical boundary conditions are often vital as, for example, in the geodynamo applications discussed later in Chapter 7, for which the liquid outer core has roughly the same electrical conductivity as the inner solid core, in contrast to the low electrical conductivity of the outer solid mantle.

Chapter 6 begins with a discussion of Alfvén waves damped by viscosity and magnetic diffusion. Inertial waves in rapidly rotating systems are introduced and the nature of their MHD extension is outlined, which is particularly pertinent to geodynamo applications. The key elements of dynamo theory, found for example in *Magnetic Field Generation in Electrically Conducting Fluids* by H. K. Moffatt, Cambridge University Press (1978), are described, together with explanations of the fundamental α and Ω-effects. The discussion includes a novel low-R_m derivation of α in terms of $\langle \boldsymbol{a} \cdot \boldsymbol{v} \rangle$, where $\boldsymbol{v} = \nabla \times \boldsymbol{a}$. The Earth's fluid core MHD, described comprehensively in the three chapters by D. Gubbins and P. H. Roberts of the book *Geomagnetism* edited by J. A. Jacobs, Academic Press (1987), is appraised. Davidson includes an interesting treatment of Taylor's constraint, which emphasizes its angular momentum content. Curiously $\alpha\Omega$-dynamo waves are discussed in the section on the geodynamo rather than the Solar dynamo, the application with which it is usually associated. The energy theorems for ideal MHD developed at the end of this chapter have had more impact on compressible MHD problems and such astrophysical applications can be found in the recent comprehensive textbook *Stellar Magnetism* by L. Mestel, Clarendon Press (1999). Nevertheless, within the confines of incompressible MHD, the flavour of the approach comes across clearly.

Chapter 7 is largely devoted to explaining the rudiments of the classical theory of three- (also two-)dimensional turbulence. The low-R_m MHD extensions, when the electrically conducting fluid is permeated by a uniform applied magnetic field $\boldsymbol{B}_0$, are considered. As in Chapter 5, the main thrust of this development is to emphasize the suppression of three-dimensional motion causing it to approach two-dimensionality. This provides a link to the earlier themes of Chapter 5. In the case of large-R_m turbulence, the possible dynamo generation of zero mean magnetic fields is argued but the author wisely avoids being drawn into the complicated topical issues concerning fast dynamos described elsewhere in *Stretch, Twist, Fold: The Fast Dynamo* by S. Childress & A. D. Gilbert, Springer (1995).

I found Part B particularly interesting in as much as it revealed to me the exciting low-R_m metallurgic applications, which are of course the author's specialty. It begins with an overview. Magnetic stirring is considered in Chapter 8; magnetic damping and the tendency towards two-dimensionality is treated in Chapter 9; while axisymmetric current driven flows are addressed in Chapter 10. Though each of these themes was introduced earlier in Chapter 5, here both their engineering applications and mathematical theory are developed more thoroughly. I particularly enjoyed the following Chapter 11 on MHD instabilities in aluminium reduction cells. Apparently the stability problem, which provides a particularly elegant application of shallow water theory, has only been solved recently, within the last decade. It is exciting to see a classic exercise in fluid mechanics being pertinent to such an important engineering problem with enormous financial implications to aluminium production. The book concludes in Chapter 12 with the underlying principles of magnetic levitation and induction heating caused by the application of high-frequency magnetic fields. Here, the key physical ingredient is the confinement of the induced electric currents within a thin skin depth close to the boundary of the liquid metal.

It is interesting to see how a book on an active research area can be rapidly overtaken by events. It is remarked in Chapter 6 that there is not yet an experimental fluid dynamo. In this context the ongoing efforts in Riga are mentioned. On p. 173, the author says 'Undaunted the Latvian scientists plan a second attempt at creating a fluid dynamo in the laboratory scale (albeit a very large laboratory!).' Concurrent with the publication of this book, the Latvian team reported a successful dynamo experiment in *Phys. Rev. Lett.* **86**, 3024–3027 (2001)!

A. M. Soward